Mathematics: analysis and approaches
Higher level
Paper 1

Topic: Counting

Candidate name

60 minutes

Instructions to candidates

• Write your name in the boxes above.
• Do not open this examination paper until instructed to do so.
• You are not permitted access to any calculator for this paper.
• Answer all questions.
• Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.
• Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
• The maximum mark for this examination paper is **[34 marks]**.

8 pages

Answer **all** questions. Answers must be written within the answer boxes provided.
Working may be continued below the lines, if necessary.

1.   [Maximum mark: 4]

Kelly is trying to make an integer with the numbers: 1, 2, 3, 4, and 5.

(a) How many 3-digit numbers with no repetitive digits can Kelly make?   [1]

(b) How many 3-digit numbers with repetitive digits can Kelly make?   [2]

(c) How many even 3-digit numbers with no repetitive digits can Kelly
     make?   [1]

2.   [Maximum mark: 6]

Prove by mathematical induction that $\displaystyle\sum_{i=1}^{n} i(i!) = (n + 1)! - 1$ for all $n \geq 0$,

$n \in \mathbf{Z}$.

3.   [Maximum mark: 6]

There are an equal number of boys and girls in Mrs. Smith's class. Three students from Mrs. Smith's class are randomly selected to be surveyed about their daily sleep time. Given that the probability of selecting three students of the same gender is 0.2. Determine the number of students in Mrs. Smith's class.

4.	[Maximum mark: 6]

One red, two identical white, and three identical blue marbles are placed in a Line. How many distinguishable ways can this be done if:

(a) the left end is a blue marble;	[2]

(b) blue marbles are at each end;	[2]

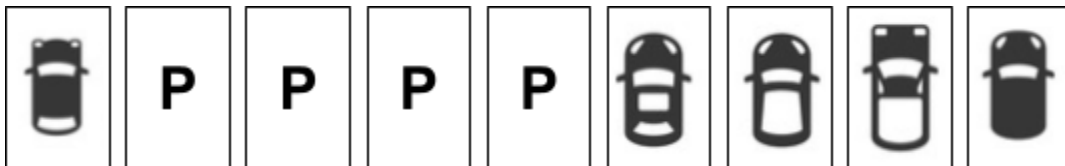(c) the left end is a blue marble and the right end is not a blue marble.	[2]

5. [Maximum mark: 4]

A shopping mall has a total of nine parking spaces, and five different cars will be parked. Find the number of distinguishable ways that the five cars can be parked if:

(a) all cars are parked together (as shown below);   [2]

(b) all vacant parking spaces are together (as shown below);   [2]

Question 5 continues on the next page

Question 5 continued

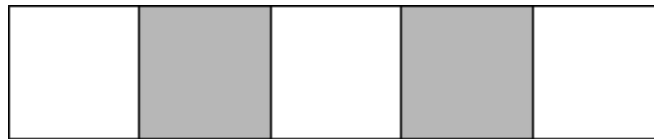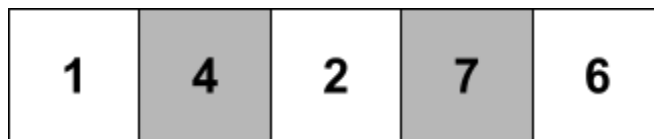**6.**   [Maximum mark: 8]

The following diagram consists of five blocks, each block will be filled with a distinct integer between 1 to 7. The number filled in a darkened block must be greater than the numbers filled in its two consecutive blocks.
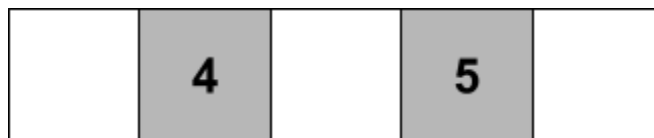

For example, 14276 is a proper number set:

(a) Find the number of different ways to fill the diagram if the two darkened blocks are filled with 4 and 5 respectively from the left to the right (as shown below).   [3]

(b) Find the number of different ways to fill the diagram.   [5]

Question 6 continues on the next page

Question 6 continued